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Letter to the Editor

Estimation of spectral density using statistical energy analysis

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1. Introduction

Dynamic responses of structures subjected to high-frequency loads are normally estimated using statistical energy analysis (SEA) developed by Lyon [1] and others. Based on the energy flow among the interconnected elements, average dynamic behaviour of the system is predicted. The response parameter averaged over the frequency band, the space as well as over an ensemble of systems is predicted. The spectral density (PSD) is normally calculated by dividing the mean square value of the predicted spatial average value of the response by the frequency band. Hence the estimated value is an average in the frequency band. This information could be sufficient in the frequency bands in which many modes are present, that is where the modal overlap is high. In frequency bands where the modal overlap is low, SEA can be applied even in such cases [2,3], the information on the frequency average value alone will not be sufficient. In such frequency bands it is necessary to estimate the peak value of the PSD. In this study a method is suggested to determine the peak value of the spectral density from the frequency averaged value of the spectral density. Acceleration responses of a plate are measured when subjected to acoustic excitation and compared with the spectral density estimated by the proposed method.

2. Estimation of spectral density

By dividing the mean square value of the predicted acceleration levels by the frequency bandwidth, the PSD can be determined. The spectral density thus obtained, called here as average PSD and denoted by ϕ, is given by

$$\phi = \langle a^2 \rangle / \delta f, \tag{1}$$

where $\langle a^2 \rangle$ is the spatial average of the mean square value of the acceleration response and δf is the frequency band.

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Consider a single degree of freedom system excited by a white noise. The mean square value of the displacement can be shown to be [4]

$$\langle x^2 \rangle = \pi\phi_{ff}/(4m^2\xi\omega_n^3), \tag{2}$$

where ϕ_{ff} is the spectral density of the force, ω_n is the natural frequency and ξ is the damping factor. The peak value of the spectral density of the displacement response, that is spectral density of the response at the natural frequency, denoted by $\phi_{xx}(\omega_n)$, is given by [4]

$$\phi_{xx}(\omega_n) = \phi_{ff}|H(\omega_n)|^2, \tag{3}$$

where $|H(\omega_n)|$ is the magnitude of the frequency response function at the natural frequency. For a single degree of freedom system magnitude of the frequency response function is given by

$$|H(\omega_n)| = 1/(2k\xi), \tag{4}$$

where k is the stiffness. From Eqs. (3) and (4)

$$\phi_{xx}(\omega_n) = \phi_{ff}/(4k^2\xi^2). \tag{5}$$

Using Eqs. (2) and (5), the mean square value of the response can be expressed in terms of the peak value of the spectral density of the response as

$$\langle x^2 \rangle = (\pi\omega_n\xi)\phi_{xx}(\omega_n). \tag{6}$$

The above equation can be modified as

$$\langle x^2 \rangle = (\pi/2)\phi_{xx}(\omega_n)\Delta, \tag{7}$$

where Δ is the half-power bandwidth. Thus, the mean square value of the response can be related to its peak value of the spectral density by using Eq. (7).

Applying these results to the modes in a frequency band and assuming that peak values of the spectral density of all the modes are the same, we get

$$\langle x^2 \rangle = N(\pi/2)\phi_{xx}(\omega_n)\Delta, \tag{8}$$

where N is the number of modes in the frequency band. If acceleration responses are considered it can be shown in a similar way that

$$\langle a^2 \rangle = N(\pi/2)\phi_{aa}(\omega_n)\Delta, \tag{9}$$

where a represents acceleration response. Therefore the peak value of the PSD of the acceleration response can be obtained from Eq. (9) as

$$\phi_{aa}(\omega_n) = \langle a^2 \rangle/\{N(\pi/2)\Delta\}. \tag{10}$$

The term $\langle a^2 \rangle$ can be estimated using SEA and the peak value of the spectral density can then be calculated using Eq. (10).

The peak PSD predicted is not the maximum value in the frequency band. Instead it assumes that all the peak values of spectral density in a frequency band have the same value. Also the predicted value is the spatial average. The method assumes that all the modes have the same damping factors in a frequency band which is anyway an assumption of SEA.

The expression derived here is valid only for frequencies where modal overlap is low. This is because the response at a frequency can not be approximated to the response of a single degree-of-freedom system when the modal overlap is high. The question remains up to what value of modal overlap Eq. (10) could be used.

Let us compare the PSD values obtained by both the methods. This can be done by comparing the denominator of the expressions for the PSD given by Eqs. (1) and (10). The denominator of the expression for peak PSD is $N(\pi/2)\Delta$ which is equal to $n(f)(\pi/2)\eta f\,\delta f$ where $n(f)$ is the modal density and η is the dissipation loss factor. The product $n(f)\eta f$ is the modal overlap. Comparing this with the denominator of Eq. (1) it can be seen that if the modal overlap is more than $2/\pi$ the peak value of the PSD predicted by Eq. (10) is lower than the average value of the PSD. This is consistent with the logic that the expression is not valid if the modal overlap is high. Hence Eq. (10) be used to determine the peak value of the PSD only for the frequencies where the modal overlap is lower than $2/\pi$. If the modal overlap is higher there is no need to determine the peak value of the PSD and the average value of the PSD will be sufficient.

3. Experimental results

A honeycomb sandwich panel having face sheets made of aluminium alloy is taken for the study. The thickness of the core is 25.4 mm. Each face sheet is having a thickness of 0.19 mm. The dimensions of the panel are 1.3×1.1 m. Measured mass of the panel is 4.3 kg and the mass without the concentrated masses is 2.75 kg. The modal density of the panel is estimated to be $0.014\,\mathrm{Hz}^{-1}$ at 315 Hz and $0.036\,\mathrm{Hz}^{-1}$ at 4000 Hz [5]. The critical frequency of the panel is estimated to be 382 Hz when the speed of sound in air is 346 m/s [6].

The panel is hung in a reverberation chamber having dimensions of $10.33 \times 8.2 \times 13.0$ m and subjected to diffuse acoustic field. The boundaries of the panel are free. The sound pressure level (SPL) is measured at three locations and the spatial average of the SPL is given in Table 1. The responses are measured at eleven randomly selected locations. Accelerometers having masses of 0.5 and 1.5 g are used. The mass loading of these accelerometers on the measured response is found to be negligible. The useful frequency range of the accelerometers is 5–8000 Hz (+/−5%).

It is to be noted that the objective of the present study is to provide a technique for estimating the peak value of the PSD from the average value of the PSD. To verify this technique one should have the average value of the PSD accurately. Otherwise the error in the average value itself can

Table 1
Estimated PSD of acceleration response of the panel

1/3 octave band centre frequency (Hz)	SPL (dB)	Average PSD (g^2/Hz)	Peak PSD (g^2/Hz)
315	122.1	0.26	0.55
400	125.6	0.47	0.74
500	122.5	0.26	0.34
630	118.2	0.080	0.088
800	112.2	0.016	
1000	112.5	0.0111	
1250	113.5	0.0113	
1600	112.8	0.0100	
2000	112.0	0.0070	

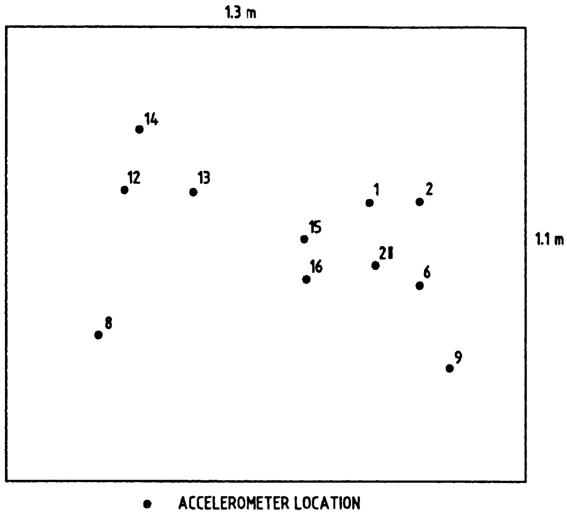

Fig. 1. Locations of accelerometers.

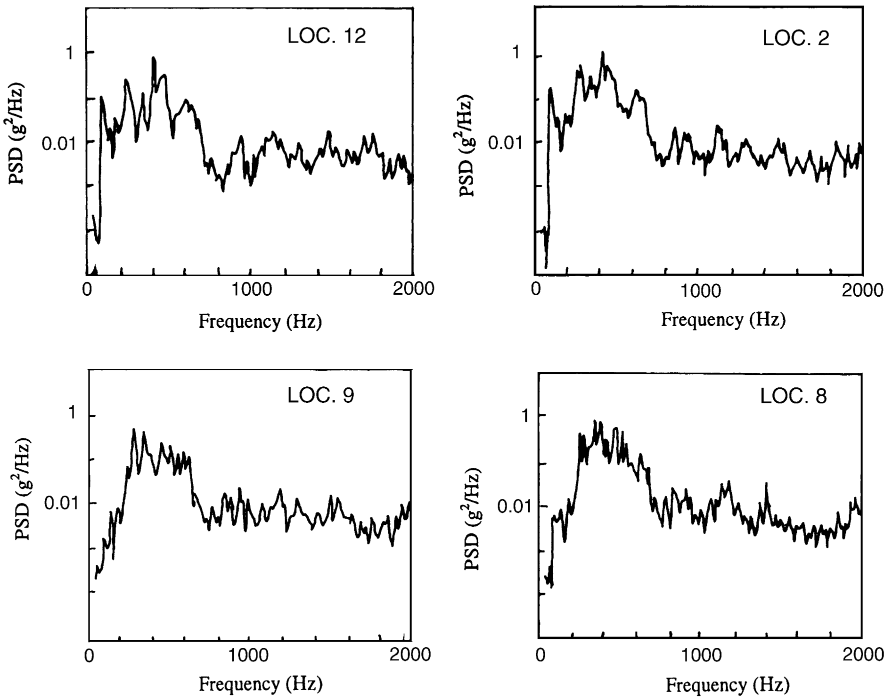

Fig. 2. PSD of the acceleration responses of the panel.

produce error in the estimation of the peak value of the PSD. For the estimation of the average PSD we require many parameters. One of them is the dissipation loss factor which can be obtained only from experimental results. Though expressions for the radiation resistance are available they are for simply supported boundaries but the present experiments are conducted with free boundaries. Instead of using some approximations it is thought prudent to use the experimental values. This helps in eliminating any error due to the use of approximate values for radiation resistance. Any error in the above two parameters can lead to errors in the estimated responses. Hence the measured values of radiation resistance [7] and dissipation loss factor values [3] are used for the calculations. Spatial average values of the measured acceleration responses show a reasonably good match with the theoretically estimated values [3].

The spectral densities of the measured acceleration responses are now determined. The PSD of the acceleration response at typical four locations are given in Fig. 2. Spectral densities are obtained with a resolution of 5 Hz upto 2000 Hz. The spectral density values are theoretically estimated using SEA and the results are given in Table 1. Eq. (1) is used to estimate the average PSD and Eq. (10) is used for estimating the peak value of the PSD. As discussed earlier Eq. (10) can be used to estimate the peak value of the PSD in the frequency range where the modal overlap is lower than $2/\pi$, in this case up to 630 Hz. For frequencies above 630 Hz only the average value is estimated. However, for frequencies above 630 Hz, information on the average value of the PSD is sufficient. It can be seen from results given in Fig. 1 and Table 1 that estimation of PSD is improved by using Eq. (10). It should be noted that the estimated peak value of the PSD is the peak value in the frequency band averaged over the space and the measured values are the PSD at different locations with 5 Hz resolution (Fig. 2).

4. Conclusions

An improved method to predict the power spectral density in the frame work of statistical energy analysis is presented. The method allows the estimation of the peak value of the power spectral density when the modal overlap is lower than $2/\pi$. The spectral densities of the measured acceleration responses of a panel subjected to diffuse acoustic field match reasonably well with the spectral density estimated using the proposed approach.

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